

Quenched Hadrons using Wilson and $O(a)$ -Improved Fermion Actions at $\beta = 6.2$

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Abstract

We present the first study of the light hadron spectrum and decay constants for quenched QCD using an $O(a)$ -improved nearest-neighbour Wilson fermion action at $\beta = 6.2$. We compare the results with those obtained using the standard Wilson fermion action, on the same set of 18 gauge field configurations of a $24^3 \times 48$ lattice. For pseudoscalar meson masses in the range 330-800 MeV, we find no significant difference between the results for the two actions. The scales obtained from the string tension and mesonic sector are consistent, but differ from that derived from baryon masses. The ratio of the pseudoscalar decay constant to the vector meson mass is roughly independent of quark mass as observed experimentally, and in approximate agreement with the measured value.

Introduction The Wilson formulation of lattice fermions leads to matrix elements which differ from their continuum counterparts by terms of order the lattice spacing, a . These corrections, which cannot be calculated reliably in perturbation theory, represent a substantial uncertainty in the determination of hadronic matrix elements. A tree-level improved theory has been proposed [1, 2], which in perturbation theory eliminates all terms of order $(g_0^2)^n a \log^n a$ [2]. Our objective, here, is to look for evidence of such improvement in masses and decay constants of the light hadrons. In this letter, we summarise the results of a study in quenched QCD on a set of 18 configurations of a $24^3 \times 48$ lattice at $\beta = 6.2$, using both the standard Wilson fermion action:

$$S_F^W = a^4 \sum_x \frac{1}{a} \left\{ \bar{q}(x)q(x) + \kappa \sum_\mu \left[\bar{q}(x)(\gamma_\mu - r)U_\mu(x)q(x+\hat{\mu}) - \bar{q}(x+\hat{\mu})(\gamma_\mu + r)U_\mu^\dagger(x)q(x) \right] \right\} \quad (1)$$

and the nearest-neighbour $O(a)$ -improved, or ‘‘clover’’, fermion action [1]:

$$S_F^C = S_F^W - irg_0\kappa \frac{a}{2} a^4 \sum_{x,\mu,\nu} \bar{q}(x)F_{\mu\nu}(x)\sigma_{\mu\nu}q(x). \quad (2)$$

$F_{\mu\nu}$ is a lattice definition of the field strength tensor, which we take to be the sum of the four untraced plaquettes in the $\mu\nu$ plane with a corner at the point x [1]:

$$F_{\mu\nu}(x) = \frac{1}{4} \sum_{\square=1}^4 \frac{1}{i2g_0a^2} \left[U_{\square\mu\nu}(x) - U_{\square\mu\nu}^\dagger(x) \right]. \quad (3)$$

Full details of the simulations will be presented elsewhere [3].

Computational Details The gauge field configurations and quark propagators were obtained using the 64 i860-node Meiko Computing Surface at Edinburgh. The full machine has 1 GByte of internal memory and, for these programs, sustains around 1.5 Gflops.

We update the SU(3) gauge fields with a cycle of 1 three-subgroup Cabibbo-Marinari heat-bath sweep followed by 5 over-relaxed sweeps. The results presented here are based on an analysis for each action of the same set of 18 configurations, starting at configuration 16800 and separated by 2400 sweeps. Propagators are calculated for $r = 1$ at $\kappa = 0.1510, 0.1520, 0.1523, 0.1526$ and 0.1529 for the Wilson action, and at $\kappa = 0.14144, 0.14226, 0.14244, 0.14262$ and 0.14280 for the clover action; the latter values were chosen to match roughly the pion masses computed in the Wilson case. We use an over-relaxed minimal residual algorithm with red-black preconditioning for propagator calculations, with point sources and sinks.

String Tension It is useful to have an estimate of the scale from a quantity which is independent of the fermion action. For this reason, we present results for the string tension obtained from generalised $R \times T$ Wilson loops, using a variational scheme comprising 40 and

28 recursive blockings [4] for the spatial paths. We obtain the potential by extrapolating the 3 : 2 and 4 : 3 T -ratio results using an estimate of the excited state contribution.

We find an acceptable fit to the static potential for $2 \leq R/a \leq 12$ with the conventional lattice Coulomb plus linear term:

$$V(R) = C - \frac{E}{R} + KR \quad (4)$$

and obtain $E = 0.274(6)$ and $Ka^2 = 0.0259(9)$, where the quoted errors are purely statistical. Using this value for the string tension to set the scale, by requiring $\sqrt{K} = 0.44$ GeV, yields $a^{-1} = 2.73(5)$ GeV. These results are in agreement with a similar analysis on 20^4 lattices at the same β -value [5, 6]. We find from a comparison with lower β results, that asymptotic scaling of the string tension does not hold at $\beta = 6.2$.

Hadron Spectrum We identify the timeslice interval 12 to 16 as a common plateau region in the effective mass plots. We obtain the amplitudes and masses of mesons (baryons) by correlated least- χ^2 fits of a single cosh (exponential) function simultaneously to the appropriate forward and backward propagators. The fits all have χ^2/dof in the range 0.3 to 3. The quoted errors are 68% confidence level bounds estimated by a bootstrap procedure. The details of our statistical analysis will be presented elsewhere [3].

The pion, ρ , nucleon and Δ mass estimates in lattice units for the two actions are given in table 1. Edinburgh plots for the two actions are given in fig. 1. The plots are broadly consistent, showing a trend towards the physical value for m_N/m_ρ with decreasing pion mass.

We now consider chiral extrapolation of the hadron masses in order to compute the lattice scale for the two actions. We perform correlated linear fits to the data at all five κ values. From the bootstrap analysis, we find that there are strong correlations between the pion masses at different κ values. The behaviour of m_π^2 as a function of $1/2\kappa$ for both actions is completely consistent with PCAC behaviour throughout the quark mass range used. From the chiral extrapolation, we obtain

$$\kappa_c = 0.15329 \pm \frac{7}{4} \text{ (Wilson)}, \quad \kappa_c = 0.14314 \pm \frac{6}{4} \text{ (clover)}. \quad (5)$$

The lattice scales obtained from correlated linear extrapolations of the other hadron masses to the chiral limit are presented in table 2. For both actions, the scales derived from m_ρ agree well with the scale from the string tension, whereas the scales from the baryon masses, whilst consistent with one another, are too low, in common with previous studies using the Wilson action. This discrepancy between the baryon and string tension scales is clearly seen in the estimate of the probability distributions for the scales obtained from the bootstrap samples, shown in figure 2.

Wilson				
κ	am_π	am_ρ	am_N	am_Δ
0.1510	0.295^{+6}_{-3}	0.377^{+11}_{-5}	0.591^{+11}_{-6}	0.647^{+19}_{-10}
0.1520	0.221^{+9}_{-3}	0.332^{+15}_{-4}	0.509^{+15}_{-7}	0.582^{+19}_{-9}
0.1523	0.195^{+9}_{-3}	0.321^{+16}_{-6}	0.480^{+16}_{-9}	0.560^{+23}_{-11}
0.1526	0.164^{+11}_{-4}	0.310^{+19}_{-11}	0.445^{+16}_{-13}	0.538^{+36}_{-18}
0.1529	0.122^{+9}_{-11}	0.298^{+39}_{-34}	0.395^{+48}_{-42}	0.533^{+116}_{-40}
0.15329^{+7}_{-4}	0.0	0.278^{+25}_{-9}	0.393^{+20}_{-16}	0.497^{+31}_{-16}
clover				
κ	am_π	am_ρ	am_N	am_Δ
0.14144	0.302^{+6}_{-4}	0.395^{+13}_{-9}	0.608^{+15}_{-8}	0.678^{+15}_{-12}
0.14226	0.217^{+8}_{-6}	0.345^{+18}_{-10}	0.495^{+30}_{-8}	0.598^{+25}_{-15}
0.14244	0.194^{+9}_{-6}	0.338^{+23}_{-13}	0.460^{+34}_{-11}	0.577^{+25}_{-22}
0.14262	0.168^{+11}_{-6}	0.329^{+30}_{-23}	0.419^{+45}_{-17}	0.565^{+33}_{-33}
0.14280	0.135^{+11}_{-6}	0.313^{+35}_{-44}	0.396^{+51}_{-38}	0.586^{+88}_{-62}
0.14314^{+6}_{-4}	0.0	0.293^{+26}_{-21}	0.376^{+42}_{-18}	0.513^{+40}_{-31}

Table 1: *Hadron masses in lattice units. The last row for each action contains the values obtained by linear extrapolation to $am_\pi = 0$.*

method	$a^{-1}(\text{GeV})$	
	Wilson	clover
string tension	2.73^{+5}_{-5}	2.73^{+5}_{-5}
m_ρ	2.77^{+9}_{-23}	2.62^{+11}_{-21}
m_N	2.39^{+10}_{-12}	2.49^{+12}_{-24}
m_Δ	2.48^{+8}_{-15}	2.40^{+15}_{-17}

Table 2: *Scales determined from different physical quantities.*

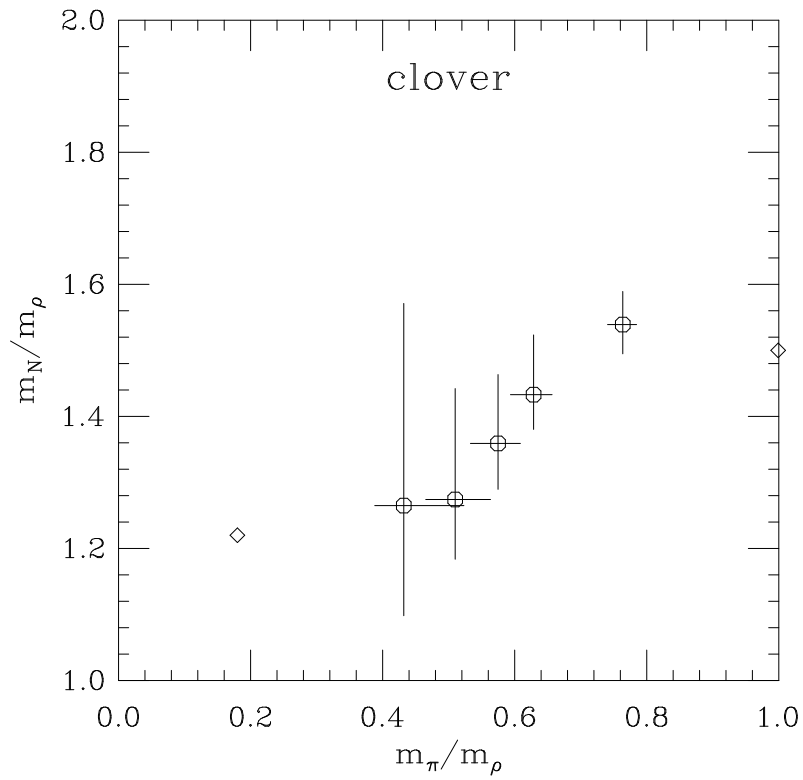
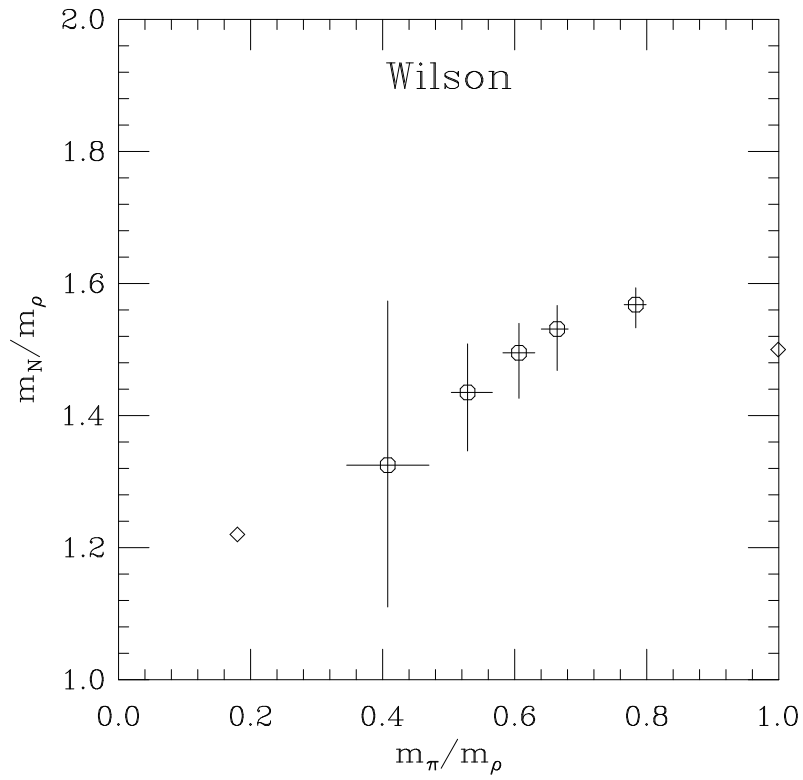


Figure 1: Edinburgh plots.

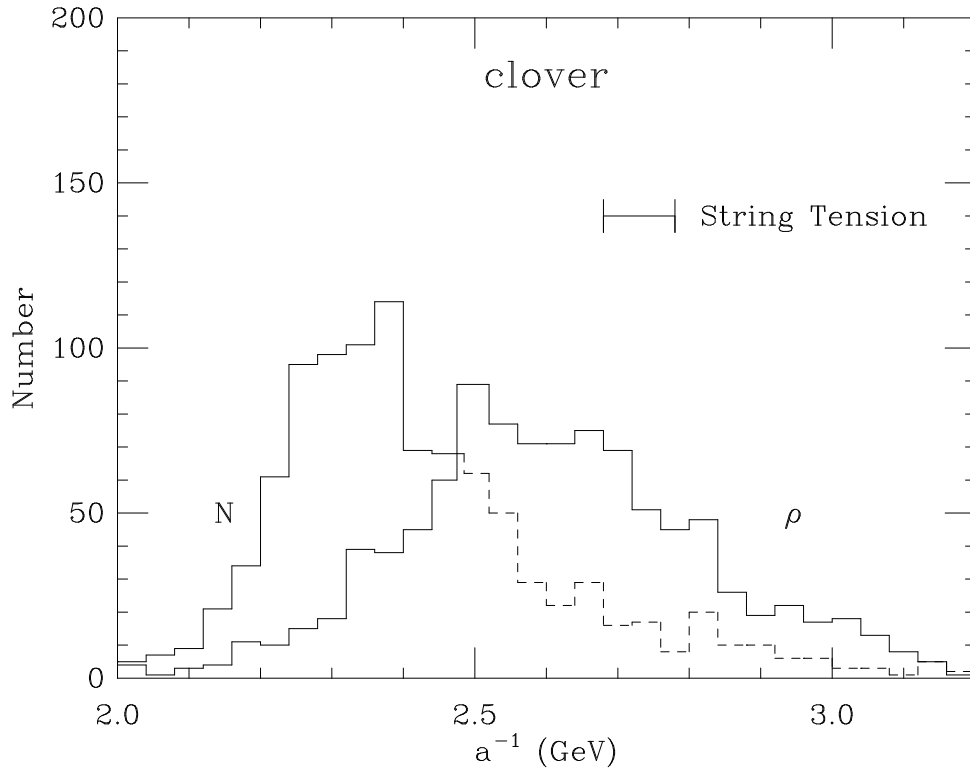
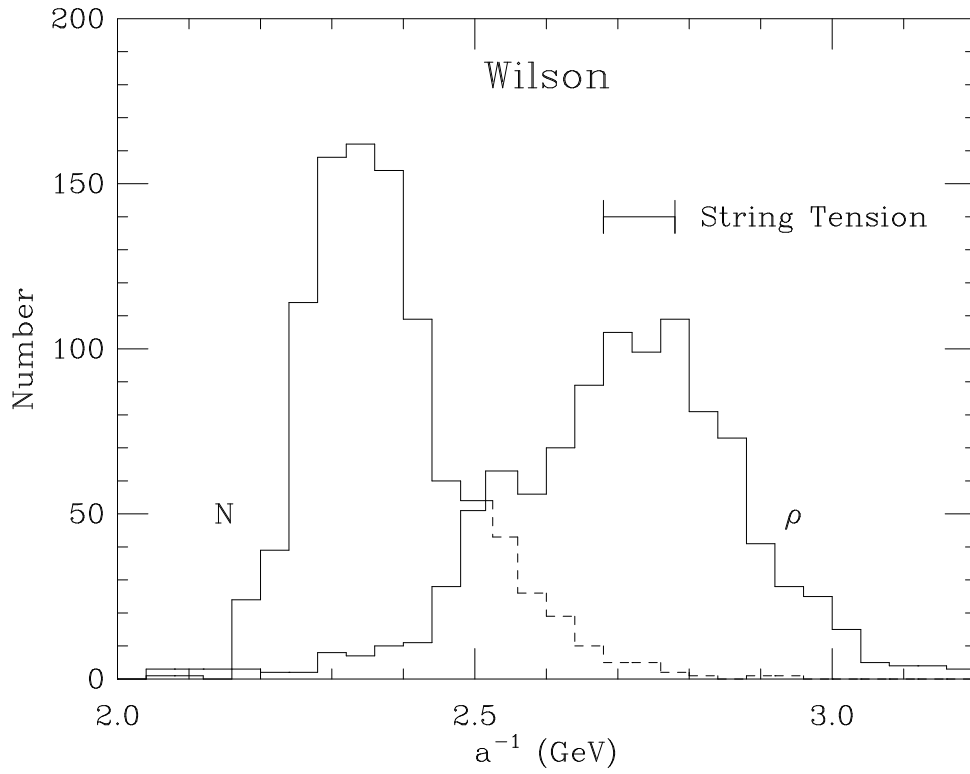


Figure 2: Distributions of the scales determined from m_ρ and m_N for 1000 bootstrap samples compared with the scale from the string tension.

In an attempt to highlight any differences arising from use of the clover action, we present estimates for the vector-pseudoscalar meson mass splitting, which in QCD-inspired quark models arises from spin interactions and therefore may be corrected at $O(a)$ by the spin term in the clover action. Experimentally, for both light-light and heavy-light mesons, $m_V^2 - m_P^2$ is very nearly independent of quark mass: 0.57 GeV^2 ($\rho - \pi$), 0.55 GeV^2 ($K^* - K$, $D^* - D$, $B^* - B$). Taking the scale from the string tension, the range of pion masses for which we have data is 330 MeV to 800 MeV. Thus, it is of interest to ask whether our data for $m_\rho^2 - m_\pi^2$, with equal-mass quarks, has similar behaviour. In figure 3, we plot the quantity $(am_\rho)^2 - (am_\pi)^2$, calculated directly from the bootstrap masses, versus $(am_\pi)^2$ for both actions. The Wilson data is consistent with previous work [7] which indicated a negative slope, inconsistent with experiment at large quark mass (e.g. for the charmonium system $m_V^2 - m_P^2 = 0.72 \text{ GeV}^2$). Apart from a constant vertical shift, the best estimates from our clover data at the four highest quark masses agree with the Wilson points; the larger statistical errors in the clover data leave open the possibility of a reduced, or zero dependence on quark mass. Using the string tension scale, the experimental value corresponds to 0.075 in lattice units, consistent with the three lightest Wilson points and all the clover points.

Meson Decay Constants The pion decay constant is defined through the matrix element of the axial current:

$$\langle 0 | \bar{q}(0) \gamma_\mu \gamma_5 q(0) | \pi(p) \rangle = f_\pi p_\mu \quad (6)$$

and our normalisation is such that the physical value is 132 MeV. In table 3, we present the values obtained for f_π and for the dimensionless ratio f_π/m_ρ , using the Wilson action with the local axial current, and the clover action with the “improved” axial current:

$$\bar{q}(x) \left(1 + \frac{ra}{2} \gamma \cdot \overleftarrow{D} \right) \gamma_\mu \gamma_5 \left(1 - \frac{ra}{2} \gamma \cdot \overrightarrow{D} \right) q(x). \quad (7)$$

Our lattice results for f_π/m_ρ vary only slowly with quark mass, in agreement with the experimental observation that f_π/m_ρ (0.17) is approximately the same as f_K/m_{K^*} (0.18).

In order to determine the physical values, the lattice results given in table 3 need to be multiplied by the appropriate renormalisation constant, Z_A^W or Z_A^C . $Z_A^W \simeq 1 - 0.132g^2$ in perturbation theory. If we use the bare coupling constant as the expansion parameter, then $Z_A^W \simeq 0.87$ and we find, for the Wilson action, $f_\pi/m_\rho \simeq 0.18 \pm \frac{2}{3}$. If instead we use the “effective coupling” proposed in reference [8], $g_{\text{eff}}^2 \simeq 1.75g_0^2$, then $Z_A^W \simeq 0.78$ and we obtain $f_\pi/m_\rho \simeq 0.16 \pm \frac{2}{3}$. The perturbative estimate [9] of the renormalisation constant Z_A^C is close to 1 ($Z_A^C \simeq 1 - 0.02g^2$); using the bare coupling leads to $Z_A^C \simeq 0.98$, whereas the effective coupling gives $Z_A^C \simeq 0.97$, yielding, for the clover action, $f_\pi/m_\rho \simeq 0.12 \pm \frac{3}{2}$ in both cases. We note that the uncertainty due to the choice of the perturbative expansion parameter is about 10% with the Wilson action and only about 1% with the clover action. Folding this

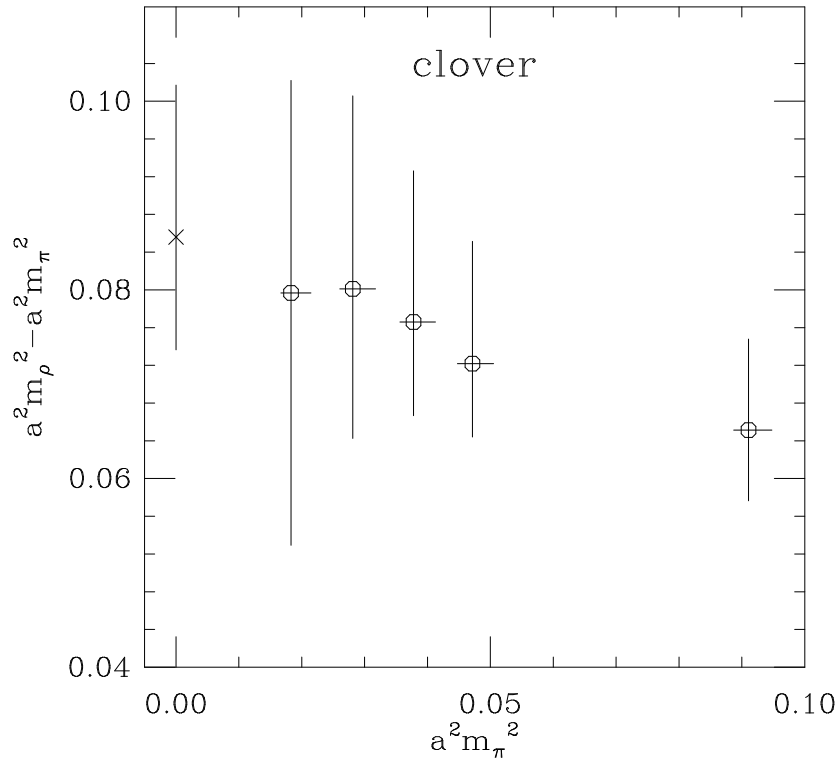
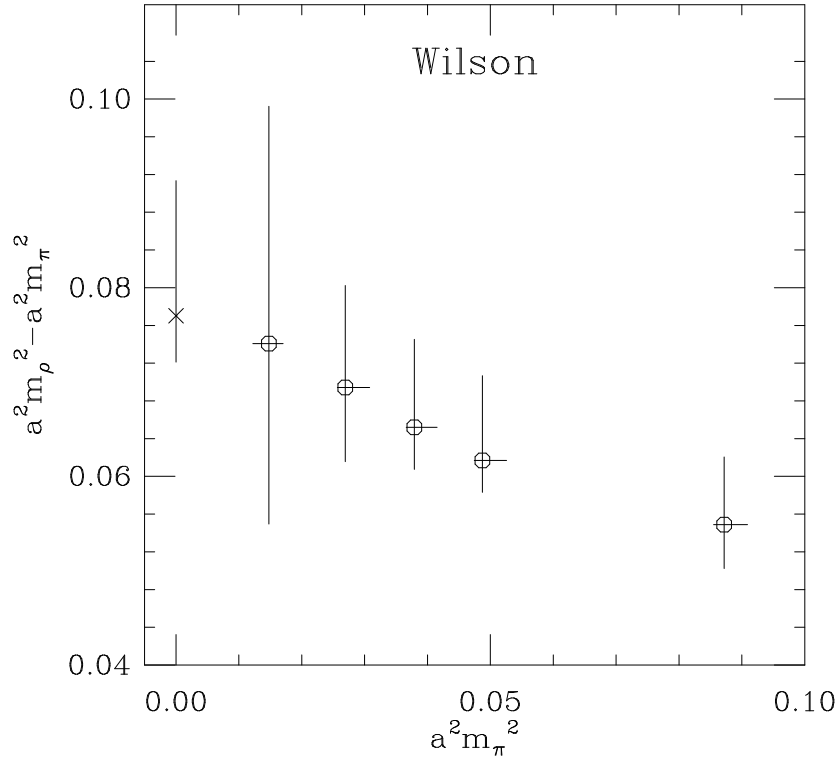


Figure 3: $a^2 m_\rho^2 - a^2 m_\pi^2$ versus $a^2 m_\pi^2$, in lattice units; the left-most point in each plot is obtained from the chiral extrapolation of the individual masses.

Wilson			clover		
κ	$(af_\pi)/Z_A^W$	$(f_\pi/m_\rho)/Z_A^W$	κ	$(af_\pi)/Z_A^C$	$(f_\pi/m_\rho)/Z_A^C$
0.1510	$0.081 \pm \frac{3}{2}$	$0.22 \pm \frac{1}{1}$	0.14144	$0.060 \pm \frac{5}{3}$	$0.15 \pm \frac{1}{1}$
0.1520	$0.069 \pm \frac{4}{5}$	$0.21 \pm \frac{1}{2}$	0.14226	$0.048 \pm \frac{7}{3}$	$0.14 \pm \frac{2}{1}$
0.1523	$0.066 \pm \frac{4}{8}$	$0.21 \pm \frac{1}{3}$	0.14244	$0.046 \pm \frac{8}{5}$	$0.14 \pm \frac{2}{2}$
0.1526	$0.065 \pm \frac{5}{11}$	$0.21 \pm \frac{2}{4}$	0.14262	$0.046 \pm \frac{8}{6}$	$0.14 \pm \frac{3}{2}$
0.1529	$0.067 \pm \frac{6}{13}$	$0.23 \pm \frac{3}{5}$	0.14280	$0.046 \pm \frac{11}{9}$	$0.15 \pm \frac{5}{3}$
0.15329	$0.057 \pm \frac{6}{8}$	$0.21 \pm \frac{2}{3}$	0.14314	$0.037 \pm \frac{10}{7}$	$0.13 \pm \frac{3}{3}$

Table 3: Values of the pion decay constant, in lattice units, and the ratio f_π/m_ρ . The last row contains values obtained by a linear extrapolation to the chiral limit.

Wilson		clover	
κ	$1/(Z_V^W f_\rho)$	κ	$1/(Z_V^C f_\rho)$
0.1510	$0.402 \pm \frac{11}{13}$	0.14144	$0.333 \pm \frac{13}{19}$
0.1520	$0.431 \pm \frac{10}{10}$	0.14226	$0.359 \pm \frac{15}{12}$
0.1523	$0.438 \pm \frac{10}{8}$	0.14244	$0.364 \pm \frac{16}{12}$
0.1526	$0.446 \pm \frac{10}{10}$	0.14262	$0.365 \pm \frac{16}{12}$
0.1529	$0.465 \pm \frac{22}{8}$	0.14280	$0.362 \pm \frac{20}{14}$
0.15329	$0.470 \pm \frac{18}{9}$	0.14314	$0.377 \pm \frac{25}{12}$

Table 4: Values of the ρ decay constant. The last row contains values obtained by a linear extrapolation to the chiral limit.

in, the overall uncertainty in the determination of f_π/m_ρ may be gauged from the bootstrap distributions in figure 4. It is apparent that, within errors, the two actions give consistent results.

In table 4, we present values of the ρ decay constant, defined by the relation

$$\langle 0|\bar{q}(0)\gamma_\mu q(0)|\rho\rangle = \frac{m_\rho^2}{f_\rho}\epsilon_\mu \quad (8)$$

where ϵ_μ is the ρ polarisation vector. For the Wilson action we use the local current and for the clover action we use the improved local current. Here, the renormalisation constant for the Wilson action is given in perturbation theory by $Z_V^W \simeq 1 - 0.174g^2$. Using the bare (effective) coupling constant, $Z_V^W \simeq 0.83$ (0.71), which leads to $1/f_\rho \simeq 0.39 \pm \frac{2}{1}$ ($0.33 \pm \frac{1}{1}$). For the clover action, $Z_V^C \simeq 0.90$ (0.83) [9] when the bare (effective) coupling constant

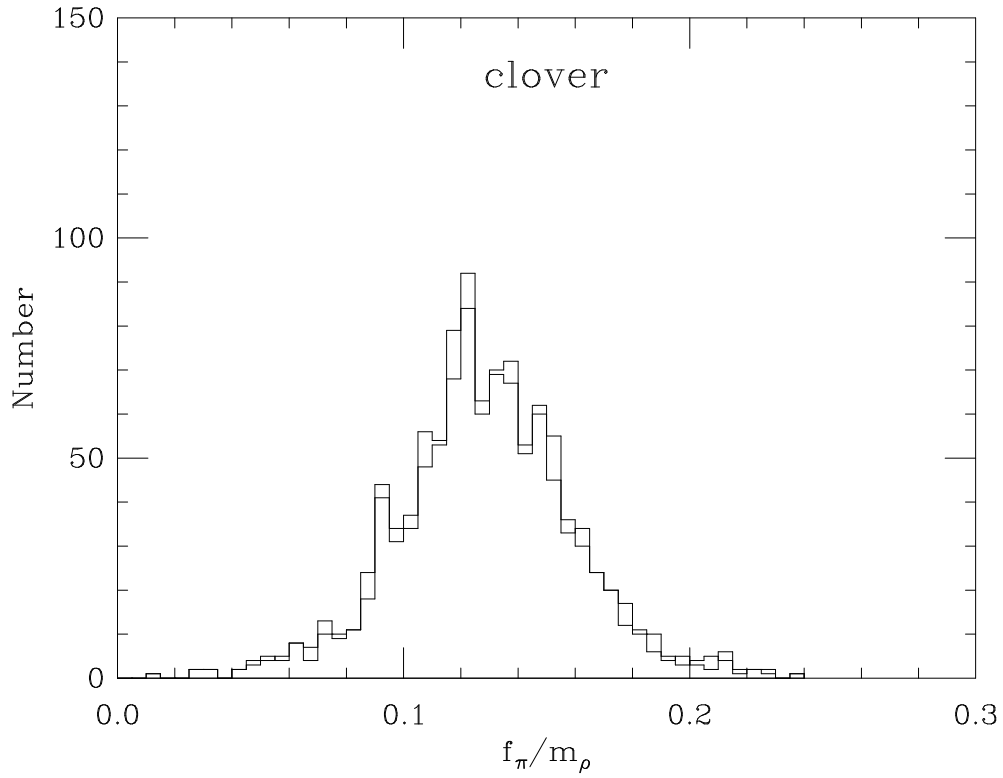
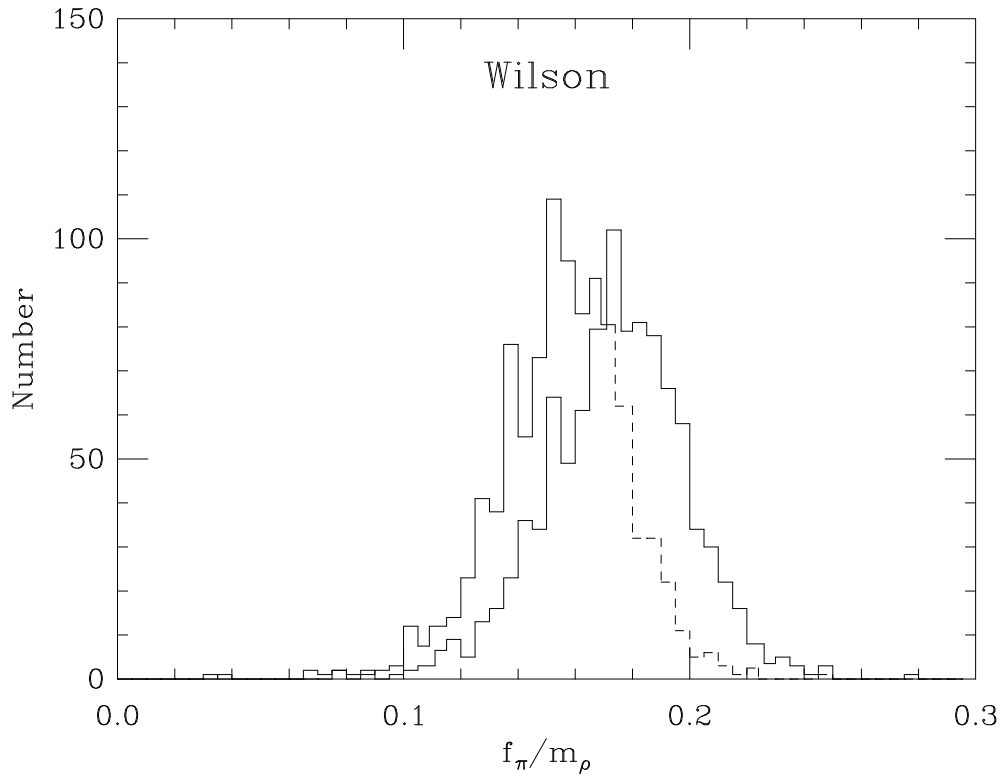


Figure 4: Distributions of f_π/m_ρ corresponding to the two choices for Z_A^W and Z_A^C described in the text, for 1000 bootstrap samples.

is used, which gives $1/f_\rho \simeq 0.34 \pm \frac{2}{1}$ ($0.31 \pm \frac{2}{1}$). The physical value of $1/f_\rho$ is 0.28(1). The uncertainty in the renormalisation constants makes it difficult to draw firm conclusions about improvement from $1/f_\rho$ computed with local currents. The perturbative uncertainty would be removed entirely by use of the conserved vector current. However, since the matrix element in equation (8) is non-forward, the use of the conserved current would not eliminate the $O(a)$ corrections in the Wilson case.

Conclusions We have presented the first study of the light hadron spectrum and decay constants for quenched QCD using an $O(a)$ -improved nearest-neighbour Wilson fermion action at $\beta = 6.2$. Having compared the results with those obtained using the standard Wilson fermion action, on the same set of 18 gauge field configurations of a $24^3 \times 48$ lattice, we see no statistically significant difference between the best estimates for quantities derived from 2-point functions, for pseudoscalar meson masses in the range 330-800 MeV. The clover data is typically noisier than the Wilson data. Nevertheless, there may yet be advantages from using the clover action for 3-point functions, where improved chiral behaviour for the B parameter of K decay has been noted [10]. We find that at $\beta = 6.2$ the scales obtained from the meson sector are consistent with that from the string tension, but there is clear evidence of an inconsistent scale from the baryon sector. f_P/m_V is roughly independent of quark mass as observed experimentally, and the numerical value is broadly consistent with the measured value.

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